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Flavour physics and CP violation

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Abstract. In this talk I give a personal selection of recent theoretical topics in flavour physics and CP violation. The main emphasis is on the theoretical methods used to calculate rates and spectra for heavy meson decays and how these results compare to the data.

PACS. 13.20.He Decays of bottom mesons

1 Introduction

Currenly flavour physics and in particular heavy flavour physics is one of the most active fields in particle physics. Large experimental efforts are made to investigate the flavour sector of the standard model, which have to be supported by theoretical progress in order to perform a precise test of our picture of flavour mixing and CP violation.

The heart of the standard-model flavour sector is the CKM matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} . \quad (1)$$

V_{CKM} is a unitary matrix carrying an irreducible phase which is the origin of CP violation in the standard model. Unitarity and the phase are usually depicted in the form of the unitarity triangle which is shown in Fig. 1. This triangle represents the unitarity relation obtained from multiplying the first column with the conjugate complex of the last column.

The area of the triangle is a measure of CP violation; hence any trivial values of the CKM angles α, β and γ ($0°$ or $180°$) would mean vanishing CP violation. It was a major breakthrough, when the B factories established a non-trivial value of β by measuring the time dependent CP asymmetry in $B \to J/\Psi K_s$ in the year 2001 [1].

It is the goal of the current experiments in flavour physics to overconstrain this triangle to test CKM unitarity as stringently as possible. Any significant inconsistency would indicate new physics in the flavour sector. However, this also requires further progress in the theoretical description of (heavy) meson weak decays to reduce the uncertanties stemming from hadronic matrix elements.

Our current understanding of flavour is in fact very unsatisfactory:

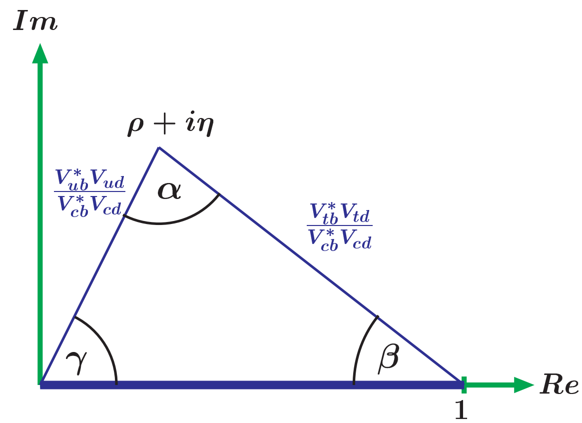

Fig. 1. The standard unitarity triangle with the definitions of ρ, η and the CKM angles α, β and γ

– A large number of parameters originates from the flavour sector. The Yukawa couplings are the source of quark-flavour mixing in the standard model, while for the leptons there is very likely an additional source of flavour mixing, which is the Majorana mass matrix. Out of the 27 parameters of the standard model (including lepton-flavour mixing assuming Majorana neutrinos) 22 are related to the flavour sector.
– The hierarchical structure of the CKM matrix elements remains a mystery.
– The quark masses (with the exception of the top) have very small values relative to the electroweak vacuum-expectation value, i.e. the Yukawa couplings are unnaturally small.
– There is no theoretical ansatz why the number of families is three.
– CP violation exhibits a specific pattern, e.g. CP violation in flavour diagonal processes (such as electric dipole moments) has not been observed; it is very small

in the standard model due to the suppression by a large number of loops [2].
– The amount of CP violation in the standard model seems to be too small to explain the observed matter-antimatter asymmetry.

I this talk I try to summarize the recent developments which will help to get closer to an answer to the above fundamental questions. The main point is to obtain a clean theoretical description of (quark) flavour physics and CP violaton. I shall focus on the field of heavy flavours and CP violation in the B system, and – even with this restriction – I shall not be able to cover all the recent topics in this field. After a short introduction I will discuss the theoretical machinery that is used to describe heavy quark decays emphasizing some of the very new ideas. In the second part I shall discuss rare decays and give a few comments on models for physics beyond the standard model.

2 Developments in heavy-flavour theory

In this section I give a mini-review of the theoretical tools (excluding lattice methods, which are discussd in a separate contribution to this conference [3]) used in describing heavy-meson decays, which are mainly based on the heavy mass expansion. I shall focus on applications where the theoretical results can be confronted with data or can be used to extract fundamental parameters.

2.1 Semi-leptonic decays, V_{cb} and V_{ub}

The main obstacle in expressing the observed flavour mixing and CP violation in terms of the fundamental parameters of the standard model is the presence of strong interactions. Hence the primary challenge for theory is to develop model-independent methods to deal with strong interactions.

As far as decays of heavy quarks are concerned, the main tool to deal with strong interactions is the heavy quark expansion (HQE) [4] and / or heavy quark effective theory (HQET) [5]. Here one makes use of the fact that the quark mass is large compared to the scale parameter of QCD Λ_{QCD}. In reality we certainly have $m_b \gg \Lambda_{\mathrm{QCD}}$, while the relation $m_c \gg \Lambda_{\mathrm{QCD}}$ can be debated.

In both HQE and HQET one performs an expansion in the quantities

$$\frac{\Lambda_{\mathrm{QCD}}}{m_Q} \quad \text{and} \quad \alpha_s(m_Q) \tag{2}$$

which can be done using methods of effective field theory and operator product expansion.

The heavy mass limit has additional symmetries beyond the ones of QCD which allow us to relate matrix elements of static heavy mesons moving with velocities v and v'. The master example is the heavy-quark-symmetry relation [5]

$$\left.\begin{array}{l}\langle B(v)|\bar{b}\Gamma c|D(v')\rangle \\ \langle B(v)|\bar{b}\Gamma c|D^*(v')\rangle\end{array}\right\} = C_\Gamma(v\cdot v')\xi(v\cdot v') + \mathcal{O}(1/m_Q, \alpha_s(m_c)) \tag{3}$$

which defines the Isgur-Wise function ξ containing all the non-perturbative information in the heavy-mass limit. The coefficient $C_\Gamma(v\cdot v')$ is calculable from heavy quark symmetries and depends on the Dirac matrix Γ in the current and on the velocities of the initial and the final state.

In the heavy mass limit the Isgur-Wise function is normalized to unity at $v = v'$; corrections to this normalizations can be discussed in HQET. In particular, for currents related to the generators of the heavy-quark symmetries Luke's theorem [6] ensures that first order ($\mathcal{O}(1/m)$) corrections to the normalization at $vv' = 1$ are absent.

These relations coming from the heavy mass limit have extensively been used to perform a determination of V_{cb} via the relation

$$\lim_{v\to v'} \frac{1}{\sqrt{(vv')^2-1}}\frac{d\Gamma}{d(vv')} = \frac{G_F^2}{4\pi^3}|V_{cb}|^2(m_B - m_{D^*})^2 m_{D^*}^3 |\xi_{A1}(vv'=1)|^2 \tag{4}$$

where $\xi_{A1}(vv')$ is one of the form factors of the axial-vector current, which becomes the Isgur-Wise function in the heavy mass limit for both the charm and the bottom quark. According to Luke's theorem $\xi_{A1}(1)$ is protected against $1/m_c$ corrections and, including $\alpha_s(m_c)^2$ [7] and an estimate of the $1/m_c^2$ corrections, the currently best estimate for this quantity is [8]

$$\xi_{A1}(vv'=1) = 0.91^{+0.03}_{-0.04} \tag{5}$$

Estimates of the $1/m_c^2$ terms are model dependent, the results from different estimates agree and are shown in the left plot of Fig. 2. The right plot shows a probability distribution suggested in [8], the broader one represents the theoretical uncertainty today, the more narrow curve is a projection into the future assuming further progress e.g. from the lattice.

The extrapolation usually uses a linear fit in which also the slope ρ^2 defined by

$$\xi_{A1}(vv') = \xi_{A1}(1)\left(1 - \rho^2[vv'-1] + \cdots\right) \tag{6}$$

is extracted. From the theoretical side the slope is restricted from unitarity and analyticity [9]. The current results from the various experiments have been collected and averaged by the heavy-flavour-averaging group [10] and are shown in Fig. 3. From this the value

$$V_{cb}^{\mathrm{excl}} = (40.2 \pm 0.9_{\mathrm{exp}} \pm 1.8_{\mathrm{theo}}) \times 10^{-3} \tag{7}$$

has been extracted [10].

The uncertainty quoted in (4) is mainly from the unknown contributions of order $1/m_c^2$ and higher and constitutes a limitation of this method unless lattice determinations will improve our knowledge of the higher order terms. First progress has been made on this, see [11].

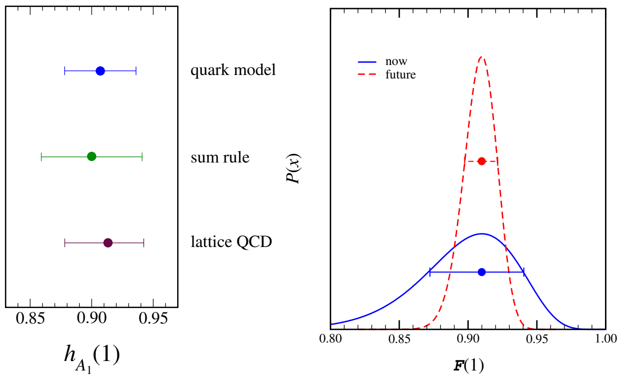

Fig. 2. Current status of the form factor $\xi_{A1} = h_{A1} = F$ in the transiton $B \to D^*$ at $v = v'$. The different entries in the left plot correspond to different methods to estimate the $1/m_c^2$ corrections. The right plot shows probability distributions for the theoretical uncertainty; the narrow curve is a projection into the future. The plot is taken from the CERN Yellow Book [8]

Another possibly more precise determination of V_{cb} can be performed using inclusive decays. In this case one performs an operator-product expansion (OPE) to write an inclusive (differential) rate as

$$d\Gamma = d\Gamma_0 + \frac{1}{m_Q} d\Gamma_1 + \frac{1}{m_Q^2} d\Gamma_2 + \frac{1}{m_Q^3} d\Gamma_3 + \cdots \qquad (8)$$

Here $d\Gamma_0$ is the partonic rate, i.e. this terms does not depend on an unknown hadronic matrix element. Due to heavy quark symmetries $d\Gamma_1$ always vanishes, while $d\Gamma_2$ can be expressed in terms of two parameters λ_1 and λ_2

$$2M_H \lambda_1 = \langle H(v)|\bar{Q}_v (iD)^2 Q_v |H(v)\rangle \qquad (9)$$

$$6M_H \lambda_2 = \langle H(v)|\bar{Q}_v \sigma_{\mu\nu} [iD^\mu, iD^\nu] Q_v |H(v)\rangle \qquad (10)$$

The contribution $d\Gamma_3$ is currently under investigation [12], the main contribution comes from the "Darwin term"

$$2M_H \rho_1 = \langle H(v)|\bar{Q}_v (iD)_\mu (ivD)(iD)^\mu Q_v |H(v)\rangle \qquad (11)$$

Applying this method to the inclusive semi-leptonic decays one uses

$$\Gamma = |V_{cb}|^2 \hat{\Gamma}_0 m_b^5(\mu)(1 + A_{ew}) A^{\mathrm{pert}}(r,\mu) \qquad (12)$$
$$\left[z_0(r) + z_2(r) \left(\frac{\lambda_1}{m_b^2}, \frac{\lambda_2}{m_b^2} \right) + \ldots \right]$$

where we only quote the structure of the formula, the details can be found in [13]. The inputs into this relation are

- **The heavy quark mass** which enters superficially at fifth power needs to be defined in a suitable scheme. It has been argued that the pole mass is not the best choice, since it suffers from a renormalon abiguity [14, 15]. A better choice is a short distance mass such as the $\overline{MS}$ mass or the kinetic mass. We will not go into any details concerning this issue; we quote only the uncertainties obtained in recent determinations [16] which is of the order of 50 MeV.

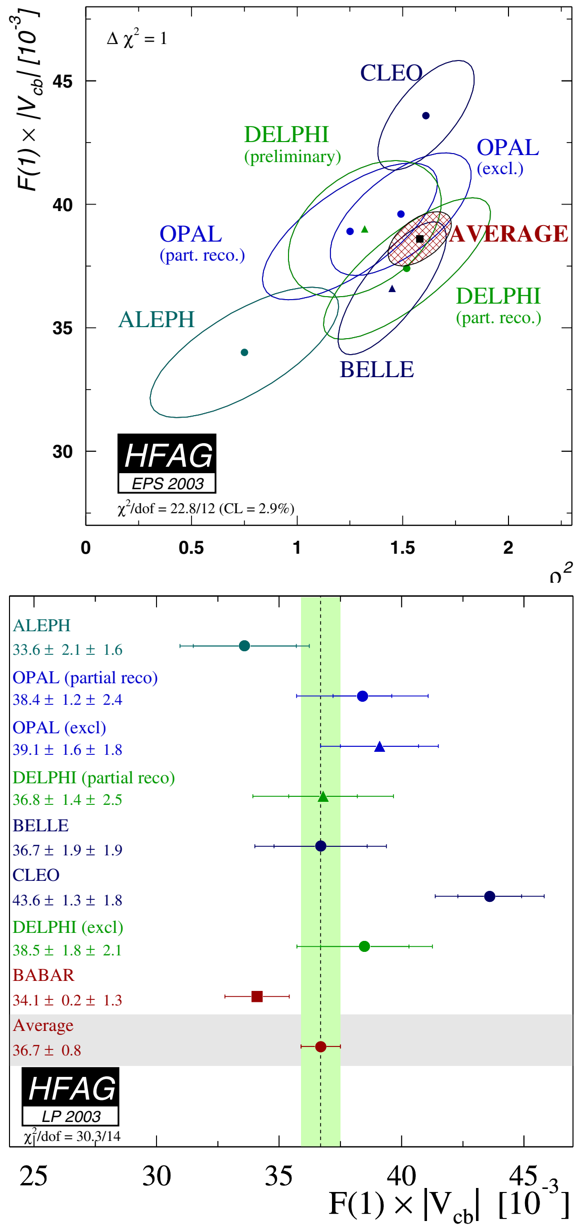

Fig. 3. Current situation of V_{cb} from different experiments. Upper plot: Correlation ellipses in the ρ^2-$\xi_{A1}(1)V_{cb}$ plane. Lower Plot: Results for $\xi_{A1}(1)V_{cb}$. The plots are taken from [10]

- **Perturbative contributions** from electroweak and QCD radiative corrections are known at $\mathcal{O}(\alpha_s)$ and for $m_c \to 0$ also at $\mathcal{O}(\alpha_s^2)$. The size of the QCD radiative corrections is strongly correlated with the definition of the mass of the b quark. In particular, a short distance mass like the kinetic mass seems to result in a good convergence of the perturbative series. Also the terms of order $\alpha_s^{n+1}\beta_0^n$ have been included [17,13].
- **The ratio m_c/m_b** enters through the phase space functions z_0 and z_2. This ratio is usually determined

from the spin averaged meson masses using

$$\overline{M}_B - \overline{M}_D = m_b - m_c - \lambda_1 \left(\frac{1}{2m_c} - \frac{1}{2m_b}\right) + \mathcal{O}(1/m_c^2) \quad (13)$$

which, however, introduces corrections as a series in inverse powers of the *charm* mass. With more data this could be avoided by fitting also the charm mass using the moments of the lepton-energy spectrum.

- **The heavy quark parameters** λ_1 and λ_2 (or μ_π^2 and μ_G^2) at order $1/m_b^2$ and ρ_1 and ρ_2 (or $\rho_{\rm D}^3$ and $\rho_{\rm LS}^3$) at order $1/m_b^3$ are determined from moments of either the lepton-energy spectrum, the hadronic mass spectrum (see below) or from the photon-energy spectrum of the radiative decay $B \to X_s\gamma$. Comparison of the different methods provides a consistency check of the HQE.

Estimating the uncertainties taking into account the correlations of e.g. the heavy-quark mass and the radiative corrections one can expect a theoretical uncertainty in this kind of determination as good as $\Delta V_{cb}/V_{cb} \sim 2\%$. The current value from the inclusive determination is [18]

$$V_{cb}^{\rm incl} = (40.8 \pm 0.9) \times 10^{-3} \quad (14)$$

It is satisfactory to note that the two results for V_{cb} from the exclusive and the inclusive determination are consistent and can be averaged. Recent concerns, that there may be still uncertainties stemming from parton-hadron duality making the naive average difficult, are probably not valid; this issue will be discussed below.

The determination of V_{ub} is complicated by the fact that the rate for $b \to u\ell\bar{\nu}_\ell$ is supressed by the square of V_{ub}. This results in a huge background from charmed decays, which has to be removed by appropriate cuts. However, this can restrict the available phase space so much that the $1/m_b$ expansion becomes poorly convergent.

There are three observables on which cuts may be imposed. The first is the energy of the charged lepton E_ℓ, which extends to larger values in the case of charmless final states. However, the expansion parameter for the lepton-energy spectrum is $1/(m_b - 2E_\ell)$ instead of $1/m_b$. While in most of the phase space $m_b - 2E_\ell$ is of order m_b, this is not true in the endpoint region $E_\ell \sim m_b/2$ which is the relevant region for the charmless semi-leptonic decays.

It has been shown that in this endpoint region one may still use the heavy mass expansion, however, one has to switch to a twist expansion [19,20,21]. In this case one proceeds similarly to deep inelastic scattering, and one is lead to define a light-cone distribution function (the analogue of the parton-distribution function in deep inelastic scattering) of the residual momentum of the heavy quark according to

$$2M_B f(\omega) = \langle B|\bar{Q}_v \delta(\omega + n_+ \cdot (iD)) Q_v|B\rangle \quad (15)$$

where n_+ is a light cone vector defined below. In the kinematic regime given by

$$p_X^2 \sim \Lambda_{\rm QCD} m_b \quad \text{and} \quad E_X \sim m_b\,, \quad (16)$$

where p_X is the momentum p_X of the final state hadrons and E_X is their energy in the rest frame of the B meson, all differential decay rates can be expressed in terms of this universal function.

In particular, for the energy spectrum of charmless semi-leptonic decays one finds in this kinematical region

$$\frac{d\Gamma}{dy} = \frac{G_F^2 m_b^5}{96\pi^3}|V_{ub}|^2 \int_{-m_b(1-y)}^{M_B - m_b} d\omega\, f(\omega) \quad y = \frac{2E_\ell}{m_b} \quad (17)$$

One may indeed obtain a model independent determination of V_{ub} by comparing this spectrum to the photon-energy spectrum of $B \to X_s\gamma$ which is directly proportional to $f(\omega)$. Although this comparison can be performed including even the subleading twist terms [22,23,24], it still needs the function $f(\omega)$ as an input, which increases the theoretical uncertainty of this method. Furthermore, only a small fraction of about 10 % of the rate is actually above the cut at $E_\ell > (M_B^2 - M_D^2)/(2M_B)$ which is needed to get rid of the charm background, so this method has certainly serious drawbacks.

The advantage of the cut on the lepton energy spectrum is that the neutrino momentum does not need to be reconstructed. However, once the neutrino momentum is known, one may also use other variables to perform cuts. One alternative is the hadronic invariant mass m_X^2 [25] which is peaked at small values for charmless decays and thus may serve as a very efficient cut. However, although in this case about 80 % of the rate are still within a cut of $M_X^2 < M_D$, there is still a dependence on the light-cone distribution function. Another alternative is to cut on the leptonic invariant mass q^2 [26], which still has about 20% of the rate within the cut of $q^2 > (M_B - M_D)^2$, however, this method does not depend on the light-cone distribution function. The effect of the cuts is schematically shown in Fig. 4. The shaded bar indicates the region which needs to be cut away in order to suppress the $b \to c$ background.

Combing these different cuts in an optimized way one can arrive at a scheme which still has about 45 % of the $b \to u\ell\bar{\nu}_\ell$ rate and only a moderate dependence on the light-cone distribution function [27]. In such a scheme a theoretical uncertainty of $\Delta V_{ub}/V_{ub} \sim 5\%$ seems to be achievable at the B factories.

2.2 Parton–Hadron duality: A potential problem of the heavy-quark expansion?

In the context of HQE a concern that has been around over the last few years is the question, if parton-hadron duality is valid in inclusive decays. This is indeed a relevant question once one is aiming at precision determinations of CKM matrix elements.

In order to discuss this question one first has to give the notion of "duality" a precise meaning. I will follow the arguments given in [28,29] and argue that very likely violations of duality will be still too small to be relevant.

The common folklore is that "sufficiently" inclusive quantites can be calculated in terms of quarks and gluons

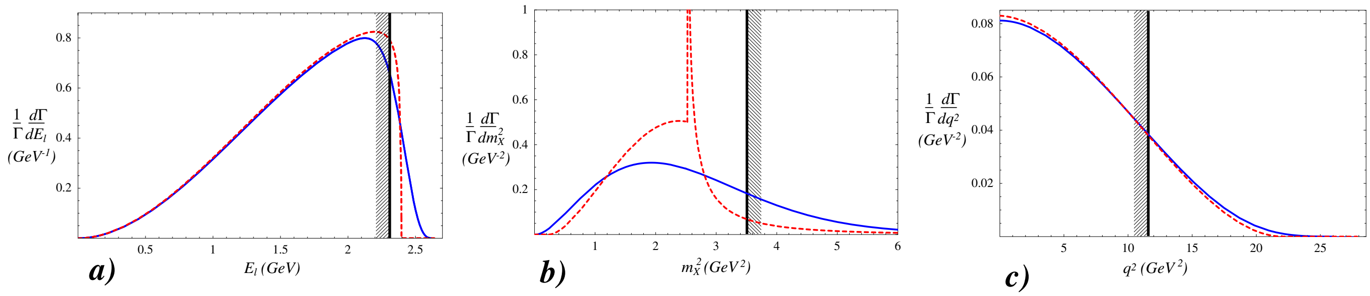

Fig. 4. Effect of the different possible cuts on $b \to u$ semileptonic decays. The region indicated by the shaded bar is contaminated by $b \to c$ decays. The solid line is the tree-level result (including the effect of the light-cone distribution function), the dashed line includes QCD radiative corrections. Plot **a** is the lepton energy spectrum, plot **b** is the hadronic invariant mass spectrum and plot **c** is the leptonic invariant mass spectrum. The plot is taken from [8]

[30]. The basis for this is the OPE, which is the $1/m$ expansion for the case at hand. Thus a precise definition of "duality violations" is that these are terms that are not described by the OPE or, likewise, render the OPE non-convergent in the same sense as the perturbative series is at best an asymptotic series.

However, to make this statement more quantitative, one would need an exact solution of QCD. Typically the OPE is performed in the euclidean region and it is known to miss exponentially small terms (such as e.g. an instanton contribution) behaving like $\exp(\sqrt{-q^2})$. However, the analytic continuation to minkowskian q^2 as needed for HQE (where $q^2 = m^2$) may turn exponentially small terms into oscillatory ones, which could have a significant effect.

In [28] various model dependent estimates have been given, which typically yield

$$\text{Duality Violations} \sim \frac{\sin(m\rho)}{m^k} \tag{18}$$

where ρ is a hadronic mass scale and k is a power which turns out to be large $k \sim 5$ in the models considered.

Thus no reliable calculation is possible on purely theoretical grounds. However, one may use data to check the validity of the OPE and of duality. One obvious possibility is to extract the HQE parameters from different sources and to check consistency; duality violations would manifest themselves in unnaturally large higher order corrections. The most recent comparisons have been shown in [31] and are reproduced in Fig. 5.

In the lower plot of Fig. 5 one can see that the extraction of the HQE parameters from the hadronic moments seems to differ slightly from the one using lepton energy moments; however, both extractions are compatible with the values obtained from $B \to X_s\gamma$, so this cannot be considered as significant.

Another observable which can be calculated reliably is the dependence of the first hadronic moments on the lower cut on the lepton energy, at least not for too high values of the cut. At the conference new data from CLEO has been presented [18] that shows a consistent picture for this observable. The data from CLEO and BaBar are shown in Fig. 6.

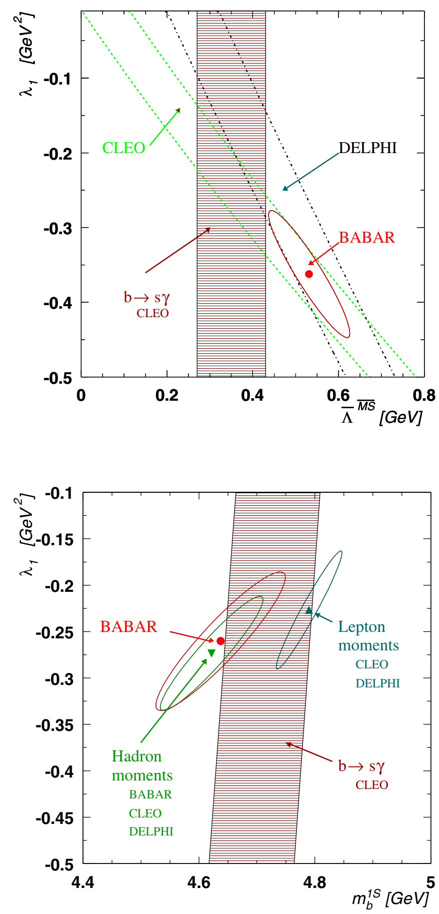

Fig. 5. Extraction of $\bar{\Lambda}$ and λ_1 from different observables. The plots are taken from U. Langeckers talk [31]

Thus concerning violations of duality there is no significant hint to a problem, the small deviations shown in Fig. 5 are statistically not significant enough to support claim that an additional uncertainty should be included due to possible violations of duality.

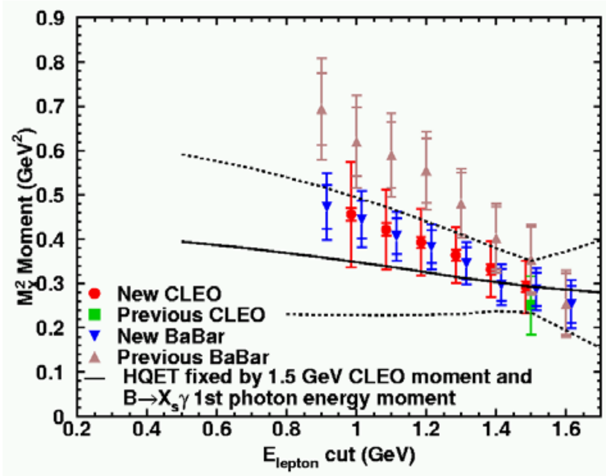

Fig. 6. Dependence of the first hadronic moment on the lower cut on the lepton energy, data versus theory. The band between the two dashed lines indicates the range of the theoretical prediction. Plot is taken from [18]

2.3 Soft collinear effective theory and QCD factorization

HQET and HQE are well suited for applications in which the light degrees of freedom are soft, i.e. with momenta of the order Λ_{QCD}. However, in heavy hadron decays we can also have the kinematic situation where the light degrees of freedom carry a large energy but have a small invariant mass. In order to discuss the point, I consider inclusive decays, but – with a few modifications – similar arguments apply to exclusive decays.

The endpoint region of inclusive heavy-to-light decays is defined by the situation where the invariant mass p^2 of the outgoing hadrons is small of the order of $\Lambda_{\mathrm{QCD}}m$, but the hadronic energy (in the rest frame of the decaying B meson) is still of the order m. Thus one has to consider

$$p^2 \sim \lambda m^2 \text{ and } (v \cdot p) \sim m \tag{19}$$

where p is the momentum of the outgoing light degrees of freedom, v is the velocity o the decaying heavy meson. and $\lambda = \Lambda_{\mathrm{QCD}}/m$. This kinematic situation is realized in inclusive decays such as $B \to X_s\gamma$ in the region where the photon energy is close to maximal, or in the endpoint region of the lepton energy spectrum of $B \to X_u \ell \bar{\nu}_\ell$, which has been discussed already above.

This particular limit can be formulated an an effective field theory, the so called soft-collinear effcetive field theory (SCET) [32]. Similar to HQET one splits off a "large" part of the momentum of the heavy quark, which is identified by using the light-cone vectors n_+ and n_-, defined by $v = (n_+ + n_-)/2$ and by a second light-like direction which is e.g. the photon momentum in $B \to X_s\gamma$ or the lepton momentum in $B \to X_u \ell \bar{\nu}_\ell$.

The momentum of the light degrees of freedom is written as

$$p = \frac{1}{2}(n_-p)n_+ + \frac{1}{2}(n_+p)n_- + p_\perp \tag{20}$$

Small invariant mass of the light degrees of freedom means that $p^2 \sim \lambda m^2$, and from $(vp) \sim m$ we infer the following power counting

$$(n_-p) \sim m_b \quad (n_+p) \sim \lambda m_b \quad p_\perp \sim \sqrt{\lambda} m_b \tag{21}$$

Without going into any details this power counting shows that one needs not only the soft degrees of freedom scaling as λ, but one has to include scales of the order $\sqrt{\lambda}$ in order to describe inclusive processes.

SCET may be formulated as an effective field theory based on a Lagrangian, implementing the above power counting. We shall not go into any details here, rather we quote the results that have been achived so far. Furthermore, in order to describe exclusive modes one has to match to a theory where the light degrees of freedom scale as $p^2 \sim \lambda^2 m^2$, which is sometimes called SCET_{II} in comparison to SCET_I discussed so far for inclusive processes. However, formulation of this theory may require to include degrees of freedom with yet another scaling behaviour which is currently under study [33].

The main result of SCET is the proof of factorization in $B \to D\pi$ [34]. It is based on the fact, that the light degrees of freedom in a decaying B meson have to be soft according to the counting scheme discussed above. Since the soft degrees of freedom can be treated exactly (using basically the non-recoil limit known from the solution of the QED infrared problem) one may remove the coupling to the other degrees of freedom by field redefinitions. In this way one may e.g. recover the the factorization of the inclusive rate $B \to X_s\gamma$ into a shape function and a (perturbatively calculable) short distance coefficient. Likewise one can show that the exclusive non-leptonic decay $B \to D\pi$ factorizes at leading order in the large mass expansion into the $B \to D$ form factor and the pion-decay constant. This statement is proven to all order in α_s using SCET. However, a poof of a similar factorization in $B \to \pi\pi$ on the basis of SCET has not yet been constructed, although first attempts exist [35]

A similar development (which actually predates SCET) is the so called QCD factorization [36], which uses the same kinematical limit but investigates the corresponding Feynman diagrams in full QCD. In this approach also the decays $B \to \pi\pi$ and $B \to K\pi$ have been investigated which are important for the determination of the CKM angle γ [37]. With this method factorization in the same sense as in $B \to D\pi$ has been shown, but up to now only on the basis of the one-loop QCD results. The typical result of factorization to leading order of the $1/m$ expansion in exclusive non-leptonic heavy-to-light transitions may be diagrammatically described as in Fig. 8.

The corresponding expression of the amplitude is

$$\begin{aligned}\langle \pi K | Q_i | B \rangle \, = \, & F_0^{B\to\pi} \, T^{\mathrm{I}}_{K,i} * f_K \Phi_K + F_0^{B\to K} \, T^{\mathrm{I}}_{\pi,i} * f_\pi \Phi_\pi \\ & + T_i^{\mathrm{II}} * f_B \Phi_B * f_K \Phi_K * f_\pi \Phi_\pi \end{aligned} \tag{22}$$

where the $T^{\mathrm{I/II}}_{M,i}$ are the hard scattering kernes which are perturbatively calculable, the $F_0^{B\to\pi/K}$ are soft non-perturbative contributions to the form factors and Φ_M are the

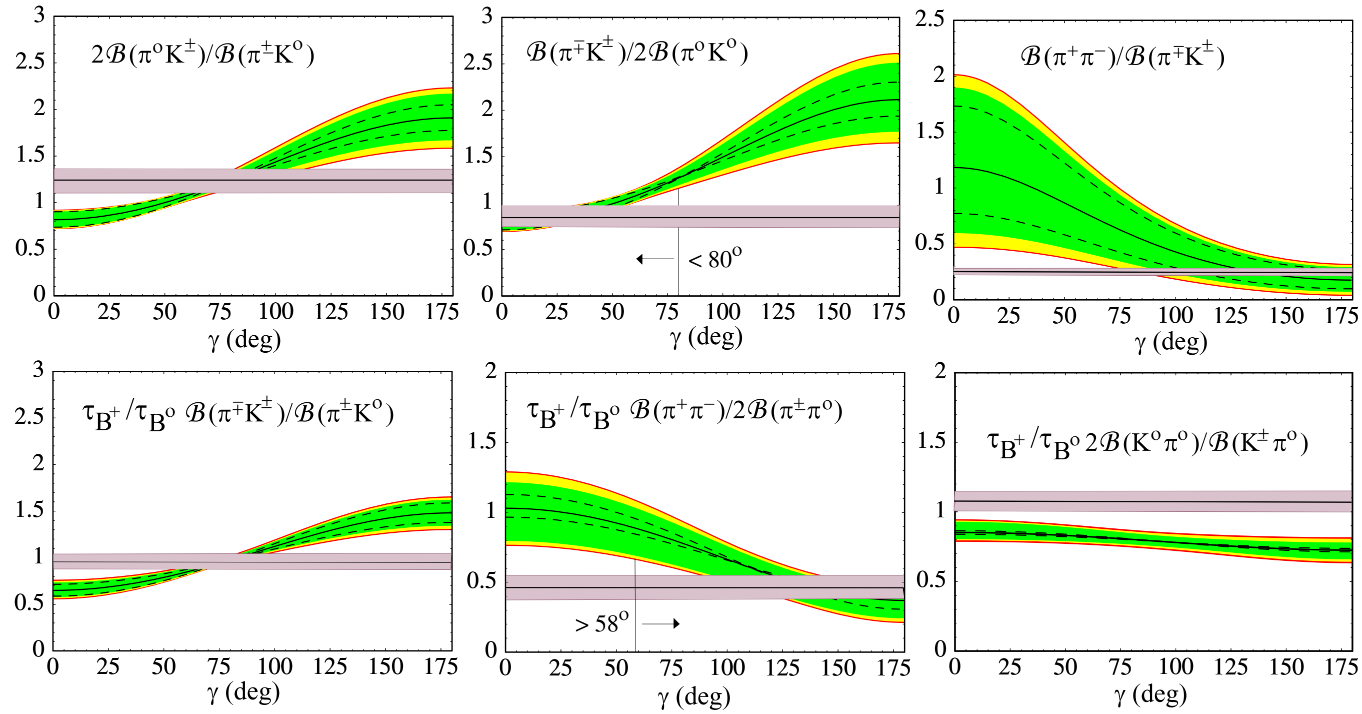

Fig. 7. Predictions of QCD factorization [37] for ratios of rates verus the CKM angle γ in comparison with data. The width of the bands indicate the theoretical and experimental uncertainties. The plot has been shown in [38, 18]

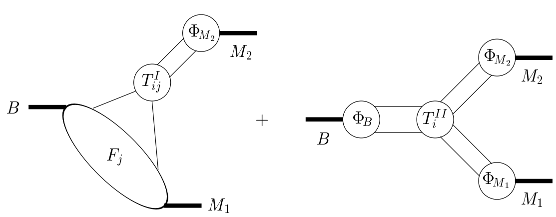

Fig. 8. Graphical interpretation of the QCD factorization theorem in charmless B decays. Figure is taken from [37]

light-cone distribution functions of the meson M, which are also non-perturbative inputs. Furthermore, the $*$ denotes a convolution, involving a light-cone variable.

Without going into any details, an important general feature of these new approaches is appearent. To leading order in the $1/m$ expansion, any imaginary part of an amplitude must originate from the hard scattering kernels $T^{\mathrm{I/II}}_{M,i}$ and thus is perturbatively calculable. From this one concludes that the strong phases entering the analysis of CP violation has to be small, namely it is either of the order $\alpha_s(m_b)$ or it is suppressed by powers of $1/m_b$. Hence the rescatting, which has been discussed intensively in connection with bounds on the CKM angle γ [39], has to be small.

QCD factorization has been applied to charmless non-leptonic B decays [37] and may now be confronted with the data, which have become quite precise. The rates of $B \to K\pi$ and $B \to \pi\pi$ decays depend on the CKM angle γ through the interference of tree and penguin contributions. Evaluating the hadronic matrix elements in QCD factorization allows us to predict the rates of these processes as a function of γ. Furthermore, one may also take ratios of rates, in which some of the theoretical uncertainties cancel.

Figure 7 shows the theoretical predictions for the ratio of branching ratios from [37] in comparison with the data. The straight shaded regions is the data published since last year. The new data show that the leading order terms of QCD factorization do not reproduce the data very well. As an example, the ratio of the $B \to K\pi$ decays of the neutral B mesons hint at a small value of the CKM angle γ, while the ratio of the neutral to charged $B \to \pi\pi$ decays hint at a large value of this CKM angle.

Furthermore, the very recently published data on $B^0 \to \pi^0\pi^0$ [40, 41] also is not compatible with the predictions of QCD factorization. The right plot in the second row involves this newly measured branching ratio, and the central value (averaged over the measurements of BaBar and Belle) is outside of the range of the theoretical predictions or any value of γ.

It is worthwhile to point out that QCD factorization and SCET are systematic approaches based on an expansion of QCD. In contrast to models, where the dependence on the specific model has to be treated like a systematic uncertainty, such systematic methods allow at least an estimate of the remaining uncertainties. In particular, QCD factorization has put the description of exclusive non-leptonic decays on a sound theoretical basis. Compared to the previously used naive factorization [42] one can

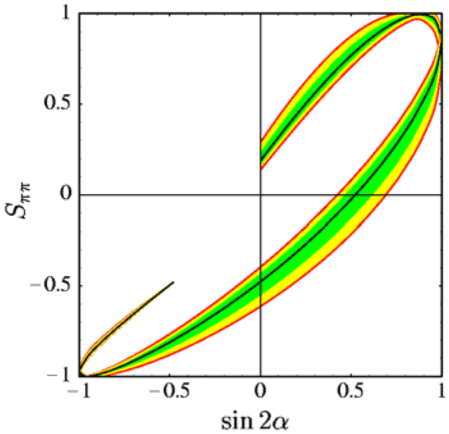

Fig. 9. Allowed region in the $S_{\pi\pi}$-$\sin 2\alpha$ plane as predicted from QCD factorization. The width of the band indicate the theoretical uncertainty. Plot taken from [37]

identify additional contributions which in general improve the description. However, the data shown above are hard to explain in any theoretical framework [43,44], and also in QCD factorization there are many sources of theoretical uncertainties that need to be understood in order to identifying the sources of possibly large corrections.

One may also use QCD factorization to predict time-dependent CP asymmetries. Here the decay $B^0 \to \pi^+\pi^-$ is the phenomenologically most interesting one, since it allows access to the CKM angle α. The time-dependent CP asymmetry is written as

$$\mathcal{A}_{\rm CP}(t) = C_{\pi\pi}\cos(\Delta m\, t) - S_{\pi\pi}\sin(\Delta m\, t) \qquad (23)$$

where the lifetime difference in the B^0 system is neglected. Note that sometimes a different notation $A_{\pi\pi}$ for the coefficient in front of the cosine term is used where $C_{\pi\pi} = -A_{\pi\pi}$.

Without penguin-tree interference one would have

$$S_{\pi\pi} = \sin(2\alpha) \quad C_{\pi\pi} = 0\,; \qquad (24)$$

however, this interference leads to deviations from (24). In QCD factorization these deviations are perturbatively calculable and lead to a prediction for $S_{\pi\pi}$ shown in Fig. 9, where the relation between $\sin 2\alpha$ and $S_{\pi\pi}$ is plotted.

The data on the CP asymmetry in $B \to \pi^+\pi^-$ is not yet conclusive. While BaBar finds a result compatible with the predictions of QCD factorization, the Belle result indicates large rescattering and sizable strong phases which cannot be fitted to QCD facorization [45]. However, the discrepancy between the two experiments is at the level of 2σ, so data has to become more precise to settle this issue. In addition, one has to see how the recent data on $B \to \pi^0\pi^0$ fit into the picture, see above.

2.4 Flavour symmetries in non-leptonic decays

In case that the subleading terms in QCD factorization and SCET turn out to be so large that these approaches are not useful in phenomenology, one has to use other methods. One approach, which is well established, is to make use of flavour symmetries such as isospin, U-spin or even full flavour-$SU(3)$. After the classical work by Gronau London and Rosner [46] to extract the CKM angle α from $B \to \pi\pi$ using isospin, a lot of work has been done using flavour symmetries. One of the most recent papers deal with the extraction of γ from the decays $B \to K\pi$ [47] which we shall discuss to contrast the results from QCD factorization for these processes.

Following [47] we consider the observables

$$\begin{matrix} R_{\rm c} \\ A_0^{\rm c} \end{matrix} \equiv 2\left[\frac{{\rm BR}(B^+ \to \pi^0 K^+) \pm {\rm BR}(B^- \to \pi^0 K^-)}{{\rm BR}(B^+ \to \pi^+ K^0) + {\rm BR}(B^- \to \pi^- \overline{K^0})}\right] \qquad (25)$$

$$\begin{matrix} R_{\rm n} \\ A_0^{\rm n} \end{matrix} \equiv \frac{1}{2}\left[\frac{{\rm BR}(B_d^0 \to \pi^- K^+) \pm {\rm BR}(\overline{B_d^0} \to \pi^+ K^-)}{{\rm BR}(B_d^0 \to \pi^0 K^0) + {\rm BR}(\overline{B_d^0} \to \pi^0 \overline{K^0})}\right] \qquad (26)$$

which are sensitive to both the CKM angle γ and the strong phase δ between the tree and the penguin contributions. Using $SU(3)$ flavour symmetry and neglecting certain rescattering terms which are believed to be small, one may relate amplitudes in $B \to K\pi$ to the ones in $B \to \pi\pi$, and thus one can study the allowed regions for the observables (25) and (26) in the R-A_0 plane.

In Fig. 10 the R-A_0 plane it is shown. For fixed values of γ one obtains elliptical curves and the shaded region represents the envelope of these curves, assuming the currently allowed region for γ. The data point indicates the current data on $B \to K\pi$ decays; the data seem to prefer a large value of γ.

Likewise one may plot the allowed ranges for different values of the strong phase δ. This is shown in Fig. 11. This analysis seems to point at sizable strong phases in contrast to the results form QCD factorization.

2.5 Results from QCD sum rules

Another well established method in the calculation of hadronic matrix elements is the approach through QCD sum rules. This methods makes use of duality and analyticity to perform analytic continuations from the deep-eucliedean region (where perturbative calculations are performed) to the minkowskian region, where the correlators are needed.

Since QCD sum rules are the only QCD based method that allows to obtain an estimate of hadronic matrix elements by an analytic calculation, an enormous ammount of work has been devoted to them. In the sector of heavy quark physics recently heavy-to-light form factors have been discussed in [48,49] and also the comparison to QCD factorization has been performed [50].

Here we shall restrict ourselves again to recent developments concerning the decays $B \to K\pi$. It has become customary to use flavour symmetries in exclusive non-leptonic decays, and the question, how well these symmetries actually hold, is very difficult to answer. QCD sum rules offer a way to at least estimate the breaking of flavour symmetries. This has been done recently for the

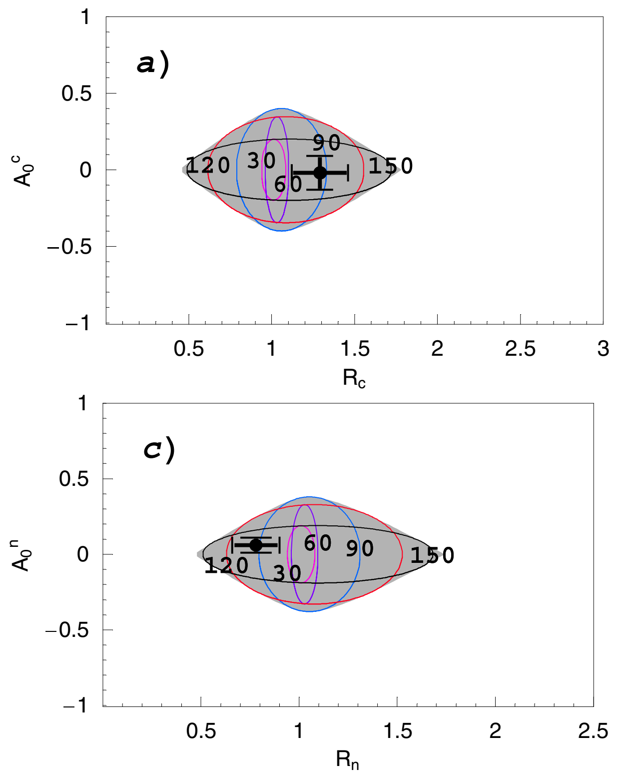

Fig. 10. Allowed region in the R-A_0 plane, assuming $SU(3)$ flavour and the standard model range for γ. The upper plot **a** is for the charged, the lower **c** for the neutral modes. The lines represent fixed values of γ. The data point is the current status of $B \to K\pi$. The plot is taken from [47]

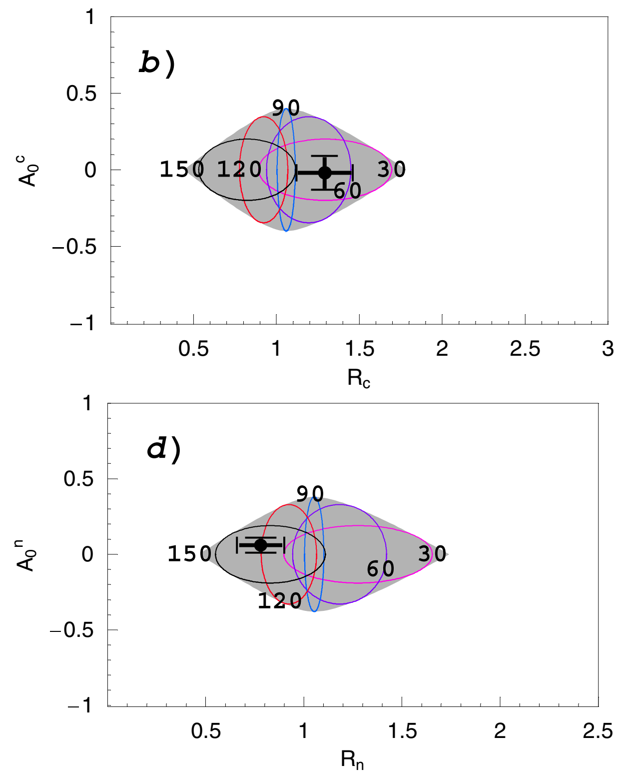

Fig. 11. Allowed region in the R-A_0 plane, assuming $SU(3)$ flavour and the standard model range for γ. The upper plot **b** is for the charged, the lower **d** for the neutral modes. The lines represent fixed values of δ. The data point is the current status of $B \to K\pi$. The plot is taken from [47]

decays $B \to K\pi$ in [51]. In QCD the origin of $SU(3)$ breaking is the strange-quark mass m_s which leads also to the fact that the value of the strange-quark condensate $\langle \bar{s}s \rangle$ is different from the value $\langle \bar{q}q \rangle$ for the up- and down quarks. However, there is no way to obtain this relation between m_s and $\langle \bar{s}s \rangle - \langle \bar{q}q \rangle$ so one keeps these two parameters independently.

One possible $SU(3)$ relation, which does not involve a matrix elements with B_s mesons, is given by

$$A(B^- \to \pi^- \bar{K}^0) + \sqrt{2} A(B^- \to \pi^0 K^-) \quad (27)$$
$$= \sqrt{2} \left(\frac{V_{us}}{V_{ud}} \right) A(B^- \to \pi^- \pi^0) \{1 + \delta_{SU(3)}\}$$

where we have introduced an $SU(3)$-breaking parameter $\delta_{SU(3)}$ which vanishes in the $SU(3)$ limit. Performing a light-cone QCD sum rule estimate of $\delta_{SU(3)}$ one obtains [51]

$$\delta_{SU(3)} = (0.210^{+0.015}_{-0.014}) - \left(0.008^{-0.015}_{+0.013}\right) i \quad (28)$$

which is of the size expected from e.g. the ratio of f_K/f_π.

However, the prediction of this quantity depends on the light-cone distribution used for the kaon. While the distribution amplitude for the pion is assumed to be the asymptotic one, the manifestation of $SU(3)$ breaking in the distribution amplitude is its asymmetry for the kaon, which is given by the coefficient a_1 of the first Gegenbauer polynomial appearing in the asymtotic expansion of the distribution amplitudes. This coefficient, however, has been recently reanalysed in QCD sum rules [52], yielding $a_1 < 0$, a somewhat counter-intuitive result, namely that the strange quark in the kaon carries a smaller momentum fraction then the light quark. In order to quote a definite number for $\delta_{SU(3)}$ this issue has to be clarified [53].

3 Rare flavour–changing neutral–current decays of B mesons

Rare flavour-changing neutral current decays of the b quark such as $b \to s\gamma$ and $b \to s\ell^+\ell^-$ are important modes to constrain new physics. Being loop processes in the standard model, it is generally assumed that these modes have a good sensitivity to "new physics" beyond the standard model.

3.1 Theoretical developments in $B \to X_s\gamma$

After the leading order effective Hamiltonian has been constructed some time ago [54], a lot of work has been

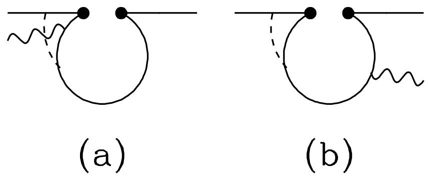

Fig. 12. Examples of diagrams leading to an m_c/m_b dependence at two loops

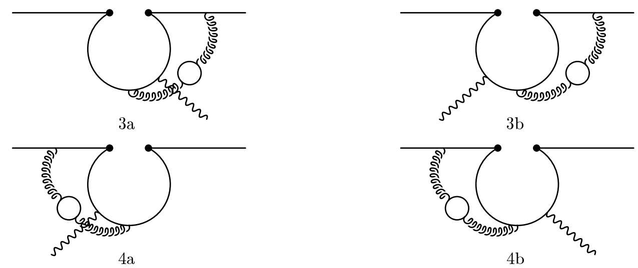

Fig. 13. Examples for three-loop n_f-dependent contributions to $B \to X_s\gamma$

done to improve the leading order result. In fact, the QCD effects in this decay turn out to be large and large logarithms have to be summed improving the $\mathcal{O}(\alpha_s)$ result. Still the residual dependence on the renormalization scale remains significant and thus the next-to-leading order terms have to be included. The various contributions have been calculated subsequently in [55,56,57,58] and the dependence on the renormalization scale has been reduced significantly.

From the experimental side this decay is interesting, since it has a reasonably large branching fraction. The first measurement by CLEO in 1995 [59] indicated that the branching ratio is indeed in the expected range. In the meantime the data has improved a lot; the experimental uncertainty has reached the level of ten percent [60].

However, it has been noted recently that the calculation of the matrix element induces a dependence on the charm-quark mass at the two-loop level [56,61]; examples for the relevant diagrams are shown in Fig. 12

The main point is that the dependence on the parameter m_c/m_b is very steep and small variations change the prediction for the branching fraction dramatically. It has been argued in [62] that one should use the pole mass for the b quark, since the b quark is an external line with the b quark in the B meson almost on-shell, while the c quark is inside a loop and hence a short distance mass like the $\overline{\mathrm{MS}}$ mass is appropriate. Of course, this is only a guess for the higher order corrections, but it indicates the range of uncertainties.

Inserting $m_c^{\overline{\mathrm{MS}}}(m_b)/m_b^{\mathrm{pole}}$ shifts the prediction for the rate by one sigma upwards compared to the values obtained from $m_c^{\mathrm{pole}}/m_b^{\mathrm{pole}}$ and hence the uncertainties from this source are significant.

In order to settle this issue one has to perform a next-to-next-to-leading order calculation, involving an anomlous dimension at the four-loop level and the calculation of the three-loop finite terms. Clearly this is a technical challenge and first steps have been performed, namely the n_f dependent terms have been investigated [63]. This involves the calculation of diagramms of the type shown in Fig. 13.

The current status of the theoretical prediction is not satisfactory; although the NLO result is fully known, the theoretical predictions in the standard model for $B \to X_s\gamma$ still vary quite a bit. Including the uncertainty from the masses discussed above, one may quote

$$\mathrm{Br}(B \to X_s\gamma) = (3-4) \times 10^{-4}\,, \tag{29}$$

which is not satifactory and the NNLO calculation is urgently needed to perform a test of the standard model using this decay.

3.2 Theoretical developments in $B \to X_s\ell^+\ell^-$

The mode $B \to X_s\ell^+\ell^-$ is complementary to the decay $B \to X_s\gamma$, since it tests different contributions to the effective Hamiltonian. Also here the leading and next-to-leading contributions have been calculated in the standard model, allowing a test for new physics.

The effective Hamiltonian relevant for these decays may be written as

$$H_{\mathrm{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu)\, O_i(\mu)\,, \tag{30}$$

$$\begin{aligned} O_{1\ldots6} &= \text{Four Quark Operators,} \\ O_7 &: b \to s\gamma \quad O_8 : b \to sg \\ O_9 &= \frac{e^2}{g_s^2}(\bar{s}_L\gamma_\mu b_L)\sum_\ell(\bar{\ell}\gamma^\mu\ell) \\ O_{10} &= \frac{e^2}{g_s^2}(\bar{s}_L\gamma_\mu b_L)\sum_\ell(\bar{\ell}\gamma^\mu\gamma_5\ell) \end{aligned} \tag{31}$$

From this we infer that $B \to X_s\gamma$ mainly tests the coefficient C_7, while $B \to X_s\ell^+\ell^-$ also tests the coefficients C_9 and C_{10}.

It has been pointed out already some time ago that a new physics contribution will manifest itself in a deviation of the coefficients from their standard-model value [64]. However, in order to disentangle the various contributions one has to consider not only the total rate, but also e.g. the differential forward-backward asymmetry defined by

$$A_{\mathrm{FB}}(s) = \int d(\cos\Theta)\mathrm{sgn}(\cos\Theta)\frac{d^2\Gamma(B \to X_s\ell^+\ell^-)}{ds\, d(\cos\Theta)} \tag{32}$$

where Θ is the angle between the ℓ^+ and the b quark in the center-of-mass frame of the two leptons.

A measured value of the rate for $B \to X_s\ell^+\ell^-$ translates into a circle in the C_9-C_{10} plane and the measured value has to lie on this circle, if no new-physics contribution is present. Figure 14 is taken from [65] and shows the

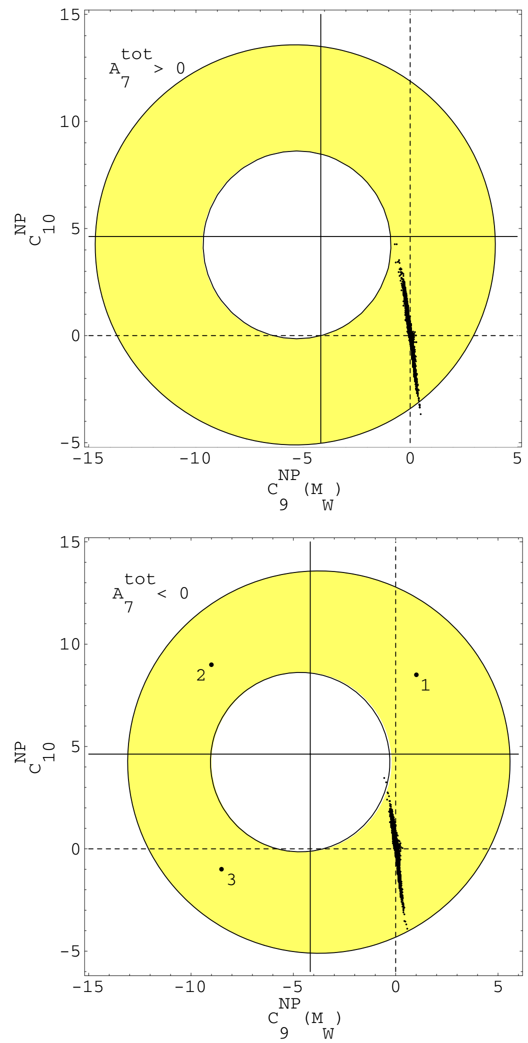

Fig. 14. Allowed region in the ΔC_9-ΔC_{10} plane, where Δ denotes the deviation from the standard model. The shaded circle is the region allowed by the current measurement of the rates. The parameter A_7 corresponds roughly to the Wilson coefficent C_7 in (30). The upper plot is for $C_7 > 0$, the lower one for $C_7 < 0$. The points around the origin are a scan over the supersymetric parameter space, keeping the $B \to X_s\gamma$ to its measured value. The figure is taken from [65]

status of such an analysis based on current data. Since $B \to X_s\gamma$ is only sensitive to the modulus of C_7, there are two possibilities for the rate of $B \to X_s\ell^+\ell^-$, since this process depends also on the sign of C_7.

In order to obtain additional information on the Wilson coefficients one can use the forward backward asymmetry (32). This observable is very sensitive to new physics contributions; in particular the standard model predicts a pronounced zero in this observable at a particular leptonic invariant mass of $s \sim 0.16m_b^2$, which has been shown to be stable under radiative corrections [66]. Figure 15 shows the forward-backward asymmetry as a function of the leptonic invariant mass. However, a measurement of the forward-backward asymmetry requires a tagging of the b quark flavour, which makes such a measurement more difficult.

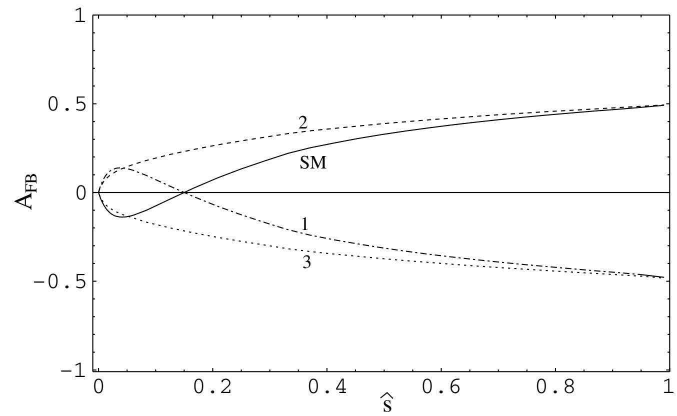

Fig. 15. Differential forward backward asymmetry as defined in (32). The solid line indicates the standard model, the dashed lines correspond to values of C_9 and C_{10} given by the points in the lower plot of Fig. 14. The figure is taken from [65]

4 Are there hints to "new physics"?

Through the current experiments the flavour sector of the standard model passes its first detailed test, and up tp now no significant discrepancy has been found indicating a deviation from the CKM picture of the standard model. On the other hand the precise extraction of the fundamental parameters in the flavour sector is often severely limited by hadronic uncertainties, reducing the sensitivity to new physics. Related to this, assigning a theoretical uncertainty to an extracted value of a fundamental parameter is often very difficult. This in turn makes the significance of a small deviation hard to judge with respect to new physics.

Compared to the gauge sector the situation in the flavour sector is much more complicated. When parametrizing non-standard-model effects in a general way by dimension-six (dim-6) operators [67], a simple and quite general parametrization in terms of e.g. the Peskin-Takeuchi parameters S, T and U [68] is possible in the gauge sector, while a similar parametrization in the flavour sector would have too many parameters to be useful. Still, with restictive assumptions one may arrive at useful statements [69,70].

4.1 Comments on models of "new physics"

The alternative is to use a definite model to describe the flavour sector. However, up to now there is no plausible model for the observed flavour structure, with the possible exception of Frogatt-Nielsen-like models [71,72].

From what has been used in the context of the gauge sector (including elektroweak symmetry breaking) and its tests performed at LEP there is a huge choice of possible models. However, generically all these models have a more complicated Higgs sector implying more parameters in the flavour sector. Even worse, due to the more complicated Higgs sector there is the possibility of having more complex couplings inducing additional CP violating phases. In this way one obtains generically far too much CP violation, if the natural size of the couplings is assumed. Furthermore, typically one also obtains sizable CP violation in flavour-diagonal processes, such as electric dipole moments.

One popular model for physics beyond the standard model is supersymetry. Inducing the breaking of supersymmetry by so-called soft-breaking terms, one obtains about 120 parameters in the flavour sector in the most general case, out of which 44 are CP violating. Thus supersymmetry clearly has a flavour problem which can only be overcome by additional assumptions.

In the most general case, a supersymmetric model has additional mixing through quark-squark interactions. This leads to a by far too large contribution to K-$\overline{K}$ mixing which has to be suppressed by a fine tuning of parameters. Likewise, one obtains large electric dipole moments e.g. for the electron, which in a generic supersymmetric model come out to be

$$d^e_{\text{SUSY}} \sim 10^{-25} e\,\text{cm} \tag{33}$$

while the current experimental limit is

$$d^e_{\text{exp}} \leq 1.6 \times 10^{-27} e\,\text{cm} \tag{34}$$

which shows the problem concerning flavour-diagonal CP violation. Clearly flavour phenomenology strongly constrains possible supersymmetric models.

One may avoid these problems by asuming supersymmetry with so-called minimal-flavour violation (MFV) [73]. Here the flavour mixing is entirely given in terms of the CKM matrix, i.e. also the quark-squark mixing is given in terms of the CKM matrix. However, there is no symmetry protecting the equality of these two mixing matrices under renormalization; furthermore, the questions concerning the CKM phases and the hierarchy of the mixing-matrix elements still remain open.

4.2 Current hints to new physics

As stated above, there are no significant hints to physics beyond the standard model. However, in B physics there are a few non-significant hints, one of which has triggered a lot of work.

In the standard model the time dependent CP asymmetries in $B \to J/\Psi K_s$ and $B \to \phi K_s$ both measure the B_d-$\overline{B}_d$ mixing phase, which is 2β. However, current data, which actually was published after the conference [76], do not yield a consistent picture. While the world average on $\sin(2\beta)$ from $B \to J/\Psi K_s$ is

$$\sin(2\beta)_{B\to J/\Psi K_s} = 0.731 \pm 0.056 \tag{35}$$

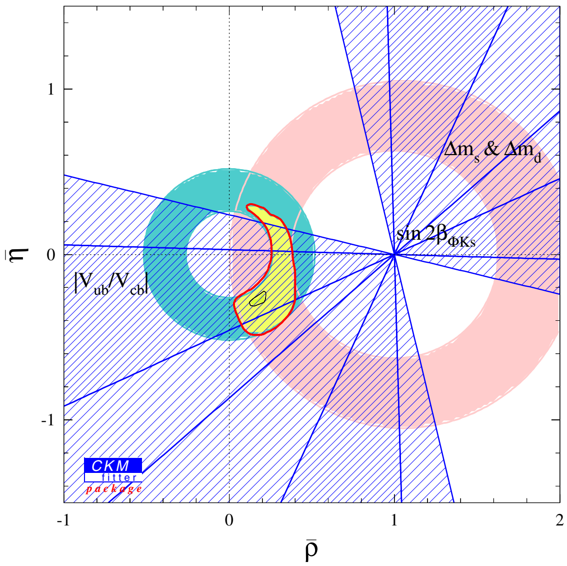

Fig. 16. CKM fit inclusing only $b \to s$ transitions and V_{ub}/Vcb. The figure is produced with CKMFitter [74] and taken from [75]

the new data from BaBar and Belle on $\sin(2\beta)$ from $B \to \phi K_s$ are

$$\sin(2\beta)_{B\to\phi K_s} = \begin{cases} -0.96 \pm 0.50^{-0.11}_{+0.09} & \text{Belle} \\ +0.45 \pm 0.43 \pm 0.07 & \text{BaBar} \end{cases} \tag{36}$$

showing a discrepancy between the two experiments at the level of two standard deviations.

If the final value is close to the Belle value (which was actually the situation at the time of the conference) this would indicate new physics, which would most probably reside in the $b \to s\bar{s}s$ piece of the effective Hamiltonian. A fit including only the observables sensitive to $b \to s$ transitions is shown in Fig. 16 and prefers the apex of the unitarity triangle in the fourth quadrant. However, from the theoretical side it is difficult to find a model in which the $b \to s\bar{s}s$ vertex is modiefied but at the same time leaves e.g. the $b \to s\gamma$ interaction at the standard-model value. However, in view of the experimental situation it is too early to draw any conclusion

5 Conclusions

The flavour sector of the standard model seems to be very well described by the CKM picture, at least as far as quarks are concerned. The overall fit based on CKM-Fitter [74] shown in Fig. 17 from G. Eigen's presentation at this conference is consistent and constrains the apex of the unitarity triangle already significantly. Based on the current data any deviation from the standard model has to be small.

The B factories as well as the second generation B physics experiments will further increase the precision of the observables. However, in order to do precision flavour

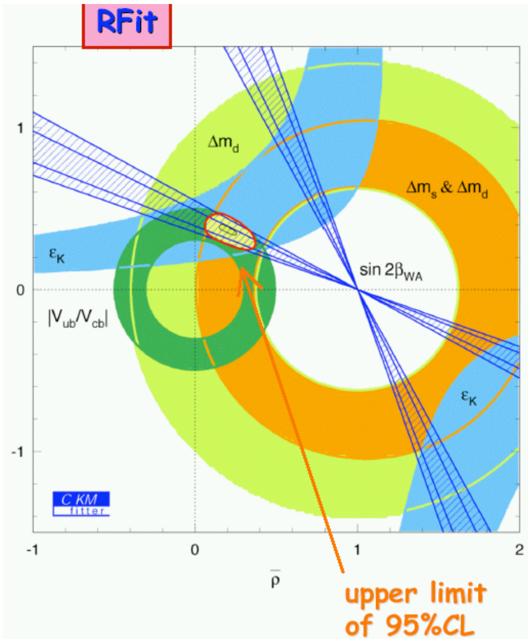

Fig. 17. The current status of the CKM fit. The plot is produced with CKMFitter [74] and taken from the presentation by G. Eigen at this conference [77]

physics one needs to improve the theoretical side further in order to match the experimental precision.

As far as semileptonic decays are concerned the situation is already quite satisfactory. The heavy quark expansion allows clean access to the CKM matrix elements V_{cb} and V_{ub}.

However, the extraction of CKM angles involves exclusive non-leptonic decays for which still no working QCD-based method exists. The problem with hadronic uncertanties is appearent since some time in the Kaon sector, where the precision of the measurement of the CP violating parameters ϵ and ϵ' in the kaon sector cannot (yet?) be matched by theoretical calculations [78].

However, the situation in B decays is better, since the b-quark mass sets a large scale, which can be used to perform an expansion. In this sense the ideas like QCD factorization and SCET are systematic approaches based on such an expansion of QCD, but the comparison with the recent data, in particular on charmless B decays, indicates that these methods still have problems, which remain to be clarified.

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